Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Connections to Other Grades

Students begin working with the Order of Operations in 3rd grade.

In 5th grade, they write and interpret numerical expressions using brackets and parentheses. They also use whole number exponents to denote powers of 10.

In 7th grade, students will solve numeric and **algebraic expressions,** equations, and inequalities with rational numbers applying the properties of operations.

In 8th grade, students will be working with integer exponents.

What is a numerical expression?

A ***numerical expression*** is a group of numbers and symbols (parentheses, operation signs, etc.) that represent a particular number; they do not contain variables or an equal sign.

SAMPLE PROBLEM

The 6th grade class has three square garden boxes to create an outdoor community garden. The lengths of the sides of the three garden boxes are 12 feet, 15 feet, and 8 feet. What is the total area needed to create the outdoor community garden?

- In the sample problem, the **numerical expression** would be: $\mathbf{12^2 + 15^2 + 8^2}$

  The expanded form of this expression would look like: 12 • 12 + 15 • 15 + 8 • 8 OR (12 • 12) + (15 • 15) + (8 • 8)

  To **evaluate the expression** means to *simplify* the numerical value of each term in the expression with attention to Order of Operations and to *compute* the answer. With the expression above, multiplication must be performed first, followed by addition of all the products.

  The evaluated form is: 144 + 225 + 64 = 433 $ft.^2$

What is a whole number exponent?

- A **whole-number exponent** is a number that tells how many times the base appears as a factor.
- The **exponent** is written in superscript to the upper right of the base number.

Remember, 6th grade students will be working with <u>*whole number*</u> exponents this year.

exponent

8^2

base

Note:
Power may be used in place of the word *exponent*.

8 • 8
"8" is the repeated factor.
"8" is called the base.
2 is the exponent because there are two repeated factors of 8.

- Say: "*eight squared, eight to the power of two, or the square of eight.*"
- Types of problems to solve: 8^2, x^2, and 4 x 4 x 4.
- Connections to Geometry ➔ 8^3: volume; 8^2: area; 8^1: line; 8^0: point.

What is an algebraic expression?

An **algebraic expression** is any group of numbers, symbols (parentheses, operation signs, etc.), and variables (letters), which represent a number. <u>They do NOT contain an equal sign</u>.

SAMPLE PROBLEM

Cards are sold for \$5 each at a team fundraiser. Weekly expenses are \$16. Write an expression for how much the team earns in a week for any number of cards sold.

The **algebraic expression** is \$5*c* - \$16 or (\$5 • *c*) - \$16

(Dollar signs are typically dropped in an expression, but they might be helpful for students to remain grounded in the context of the problem.)

Reading an Algebraic Expression: It is important to help students make the connection that a variable represents a number. Encourage students to substitute "some number" for a variable as they develop variable language.

- 36 + *p*: *thirty-six plus some numbe; 36 plus p*
- 14 • *n* or 14*n*: *fourteen times some number; 14 times n*
- $\frac{m}{8}$; *m* ÷ 8: *some number divided by eight;* m *divided by 8*
- 12(8 – *z*): *twelve times the quantity of the difference between 8 and a number; the quantity of eight minus z, times 12*

Mathematical Terms

- Letters have historically been used to represent abbreviated words (*m* means meter, *i* means imaginary numbers) or *pi* for 3.1415…
- Variables represent a quantity, so the four operations function in the same manner as with numbers. It is helpful to guide students when using letters to explicitly define the unit the variable represents in a word problem. For example, if a student is representing "Trevor's age", a variable choice might be "t" rather than "ta" because multiple variables represent multiple quantities.

VARIABLES CAN….

…represent specific numbers such as *x* + 2 = 7 where *x* can only be 5.

…be used to make statements that include a range of numbers such as "dolls cost \$4.99 (and up) at the Super Mart" represented as x ≥ 4.99 where *x* represents possible doll costs. (See Unit of Study 6)

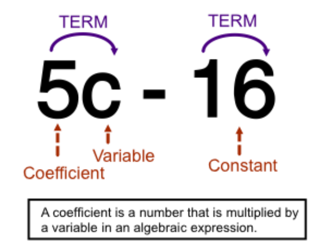
…make a statement that is true for all numbers such as *a* + *b* = *b* + *a*. The Properties of Operations fit in this category, but they are not exclusive members. Consider *a* + 2 = *y* or A=*lw*.

Have you ever heard the phrase, "Combine like *terms* to simplify the expression?" Term is an important word in mathematics and often misunderstood by students. **A term is an expression, which includes numbers and variables (sometimes containing grouping symbols) separated by addition or subtraction signs.** <u>*An expression contains three basic building blocks: numbers and variables, grouping symbols, and operation signs.*</u> Students should be able to identify the parts of an algebraic expression including variables, factors, and coefficients and the results of the operations like the sum, difference, product, and quotient. The Sample Problem expression from above, \$5c - \$16, demonstrates two **terms**.

⇒ FIRST TERM: 5*c* is viewed as a single entity or *term.*
  - 5*c* is a **variable** term, where *c* is the variable. The **term** 5*c* represents 5 groups of *c*: (*c* + *c* + *c* + *c* + *c*) or 5 • *c*.
  - 5 is the **coefficient** of the **variable "*c*",** which tells how many of them you have.

⇒ SECOND TERM: 16
  - 16 is the **constant** *(knowing the word "constant" is not in the core)*. Constants can be a little tricky, but in this instance 16 is a constant.

⇒ EVALUATING THE EXPRESSION:
  - Solve 5*c* by multiplying the coefficient and variable. If *c* = 13, then 5 × 13 = 65.
  - Subtract the second term from the first term.
    - 65 – 16 = 49



Examples of expressions with one term:	*Examples of expressions with two terms:*
2x	*2x – 5*
a² → a^2	*n + 3*
n	*ab + 3*

Evaluating Expressions and Equations using Conventional Formulas

Real-world formulas (volume, area, etc.) present an excellent way to connect exponents and variables to concrete situations found in 6th grade Geometry. As students construct and deconstruct shapes concretely and pictorially, they are able to decipher the language of conventional formulas by connecting variables to the meaning found within the context. For example, the variables used have meaning within the formula ("V" represents volume and "*s*" represents side). These types of expressions and equations can be thought of as true statements because any number can be substituted in the expression and the equation will always be true.

Equation for finding the volume of a cube

V represents "volume"
s represents "side"

$$V = s^3$$

Students develop an understanding of volume as "filling" during their work in Geometry. In terms of the equation, it is especially important to connect the meaning of the variable as it relates to filling an object. Sides are cubed because it takes three measurements to create a three-dimensional object.

Equation for finding the surface area of a cube

A represents "area"
s represents "side"

$$A = 6s^2$$

Students develop an understanding of area as "covering" during their work in Geometry. In terms of the equation, it is important to connect the meaning of the variables as it relates to "covering" a region. Sides are squared because it takes two measurements to create a two-dimensional shape. The area for one side is multiplied by 6 because there are six congruent sides on a cube.

Evaluating Expressions and Equations Using the Order of Operations

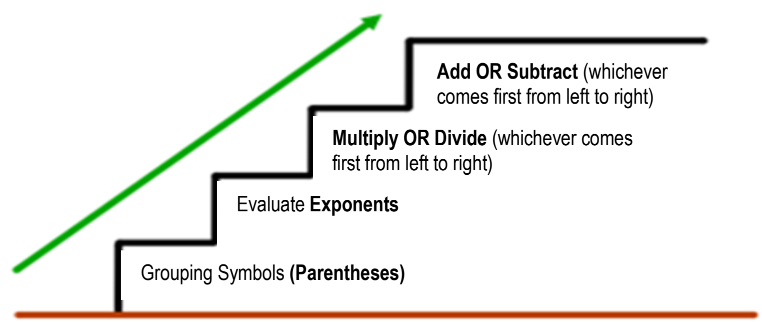
The Order of Operations is a set of rules regulating how operations should be performed in an expression or equation. It allows mathematicians around the world to reach the same conclusion when simplifying or solving a problem. They should not be confused with the Properties of Operations, which spell out the unique characteristics and relationships between and within operations.

CAUTIONARY NOTE!

PEMDAS (parentheses, exponents, multiply, divide, add, subtract) is a common mnemonic device often used in the upper grades that can lead to some confusion. Because multiply and divide are listed in a hierarchal order, it is easy for students to assume that multiplication is first. However, multiplication or division is implemented based on which operation comes first from left to right. The same pitfall can occur with addition and subtraction. When students begin operating with integers in 7th grade, it can particularly tricky. To illustrate this point, consider the following expression: *n* - 6 + 7. If 6 + 7 is solved first, the solution is *n* – 13. Using the Order of Operations, "subtract 6 and add 7" or "-6 + 7" is completed, and the correct solution is *n* + 1. Rather than knowing PEMDAS, students should recognize that there are four steps in the Order of Operations.

$$y = n - 6 + 7 \rightarrow y = n \underline{- 6 + 7} \rightarrow y = n + 1$$

$$-6 + (+7) = +1$$



Order of Operations

1 – Complete operations within grouping symbols: ( ), [ ]
2 – Evaluate exponents: x^2
3 – Multiply or divide from left to right (whichever comes first)
4 – Add or subtract from left to right (whichever comes first)

Equivalent Expressions & Properties of Operations

Commutative Property of Addition: The order of the addends does not change the sum. Example of Equivalent Expressions: $3n + 4n = 4n + 3n$.

$$a + b = b + a$$

Commutative Property of Multiplication: The order of the factors does not change the product. Example of Equivalent Expressions: $3n \bullet 6n = 6n \bullet 3n$

$$a \bullet b = b \bullet a \text{ or } ab = ba$$

Subtraction and Division are not Commutative

$$13 - 7 \neq 7 - 13$$
$$6 \neq -6$$

$$24 \div 6 \neq 6 \div 24$$
$$4 \neq 0.25$$

Associative Property of Addition: The sum stays the same regardless of how the addends are grouped using parentheses. $2a + (3a + 7a)$ and $(2a + 3a) + 7a$ are equivalent expressions.

$$(2a + 3a) + 7a = 2a + (3a + 7a)$$
$$5a + 7a = 2a + 10a$$
$$12a = 12a$$

$$(a + b) + c = a + (b + c)$$

Associative Property of Multiplication: The product stays the same regardless of how the factors are grouped.

$$(3a \bullet 4b) \bullet 5 = 3a \bullet (4b \bullet 5)$$
$$12ab \bullet 5 = 3a \bullet 20b$$
$$60ab = 60ab$$

$$(a \bullet b) \bullet c = a \bullet (b \bullet c)$$

Subtraction and Division are not Associative

$$(24 - 8) - 5 \neq 24 - (8 - 5)$$
$$16 - 5 \neq 24 - 3$$
$$11 \neq 21$$

$$(24 \div 8) \div 4 \neq 24 \div (8 \div 4)$$
$$3 \div 4 \neq 24 \div 2$$
$$\tfrac{3}{4} \neq 12$$

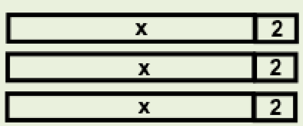
Distributive Property of Multiplication over Addition: A factor can be broken up or "distributed" to two or more different addends.

$3(x + 2) = 3x + 6$ $\quad$ $a(bc) = (ab) + (ac)$

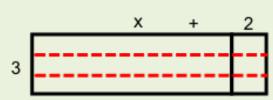
The Distributive Property is important in this situation because $x + 2$ cannot be simplified. This property would not be as useful with 3(2 + 5), which is 3(7) or 21, since the quantity inside the parentheses can be simplified.

Repeated Addition Used in an Area Model

This problem can be thought of as 3 groups of x + 2.
It might look like x + 2 + x + 2 + x + 2 (with or without grouping symbols).
The model shows the 3 being distributed to the x's and groups of 2.

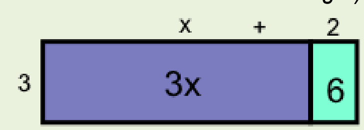


The multiple arrays can be grouped together to create one array once students understand the concept.



Moving from Repeated Addition to the Open Array Area Model

(Students have access to this model because of their previous work with the multiplication table. One factor is recorded on the horizontal edge and the other factor is recorded on the vertical edge.)



Equivalent Expressions

$3(x + 2)$
$(x + 2) + (x + 2) + (x + 2)$
$3x + 6$

Distributive Property of Multiplication over Addition *continued*

Sometimes students encounter an expression in which common factors must be determined in order to write an equivalent expression.

$24x + 18y$

Step 1: Determine the factors

$24x = 2 \bullet 2 \bullet 2 \bullet 3 \bullet x$

$18y = 2 \bullet 3 \bullet 3 \bullet y$

The common factor for 24 and 18 is 6 since $2 \bullet 3 = 6$.

$24x = 2 \bullet 2 \bullet 2 \bullet 3 \bullet x$

$18y = 2 \bullet 3 \bullet 3 \bullet y$

Step 2: Place the common factor outside the parentheses and calculate the remaining factors.

$6\,(2 \bullet 2 \bullet x + 3 \bullet y)$

$6\,(4x + 3y)$

Step 3: Identify expressions that are equivalent.

$6\,(2 \bullet 2 \bullet x + 3 \bullet y) = 6\,(4x + 3y)$

$6\,(4x + 3y) = 24x + 18y$

Using the Properties of Operations to Justify $y + y + y = 3y$

There are several ways to think about this. In the most simplistic way, one *y* plus one *y* plus one *y* equals 3 *y*'s. In essence, there is an implied "1" before each *y*. This equation reads $1y + 1y + 1y = 3y$, which certainly helps students make sense of the equivalence between the two expressions. It can also be reasoned using the Distributive and Commutative Properties and the Multiplicative Identity (any number times 1 is that number).

First, use the Multiplicative Identity to rewrite the expression: $y + y + y = y \bullet 1 + y \bullet 1 + y \bullet 1$

Next, use the Distributive Property to rearrange the factors: $y \bullet 1 + y \bullet 1 + y \bullet 1 = y(1 + 1 + 1)$

(Note: You may also distribute the 1 instead of the y as in 1(y + y + y)

Then, complete the computation on the Distributive Property: $y(1 + 1 + 1) = y \bullet 3$

Finally, use the Commutative Property to change the order of the factors: $y \bullet 3 = 3y$