CLUSTER 6: Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Uses of Variables and Connections to Other Grades

Elementary students will be exposed to three types of variable use in the Common Core. These uses span many grade levels and are a rather significant tool for building algebraic reasoning and readiness for secondary mathematics.

1) Variables stand for unknown, but specific, numbers such $x + 2 = 7$ where x can only be 5.	**2) Variables are used to make a statement that is true for all numbers such as $a + b = b + a$. The Properties of Operations fit in this category, but they are not exclusive members. Consider $y = 2 + a$ or $A=lw$.**	**3) Variables are used to make statements that include a range of numbers such as "dolls cost $4.99 and up at the Super Mart" represented as $x \geq 4.99$ where x represents possible doll costs.**
***Connections to Other Grades**: Students work with symbols for unknowns with addition and subtraction in 2nd grade. (e.g. 2 + □ = 5, □ + 2 = 5, or 2 + 3 = □) By 3rd grade, they solve for unknowns in multiplication and division. Additionally, they are introduced to algebraic language by determining what unknown number will make the solution true. (e.g. 8 x ? = 48, 5 = □ ÷ 3, or 6 x 6 = ?)*	***Connections to Other Grades**: Students reason mathematically with the Commutative and Associative Properties of Addition in 1st grade. By 3rd grade, they reason using the Commutative and Associative Properties of Multiplication and the Distributive Property (area of rectangles). In 4th and 5th grades, they work with multiplicative comparisons. For example, "Braden has 3 times as many soccer balls as Ben", expressed as 3 x n (n represents the number of soccer balls Ben has and 3 x n represents the number Braden has). They also generate equations given a rule such as y = a + 2 and work with volume formulas (V = l x w x h and V = b x h).*	***Connections to Other Grades**: This use of a variable is a new concept in 6th grade.*

Defining Equations and Inequalities

What is an Equation? An equation is a numerical statement in which the quantity on the right side of the equal sign is the same as the quantity on the left side of the equal sign. One expression **= (is equal)** to the other.	***What is an INEQUALITY?*** An inequality expresses a relationship between expressions. Those relationships come in five forms. One expression **≠ (is not equal)** to the other. One expression is **> (greater than)** the other. One expression is **≥ (greater than or equal)** to the other. One expression is **< (less than)** the other. One expression is **≤ (less than or equal)** to the other. NOTE: Multiple relationships can be shown as in $5 \leq b \leq 8$.

SYMBOL: =

These symbols do not imply an action in the way that operation signs might. They are symbols for expressing relationships between expressions.

SYMBOLS: ≠, <, ≤, >, ≥

What is a non-negative rational number?

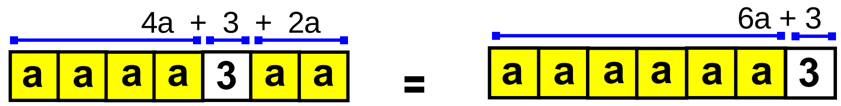
Rational numbers are Real numbers that can be expressed as a ratio of two integers (can be expressed as a fraction).
e.g. 5, -8, 0, 1/3, 0.14, -1.66, etc.

Non-negative numbers are numbers that are not negative.
↓

Therefore, non-negative rational numbers are limited to positive rational numbers.
e.g. 5, 0, 1/3, 0.14, etc.

What does it mean when an equation or inequality is true?

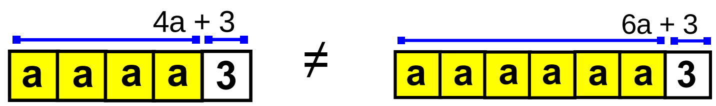
An equation is true when it states a correct relationship. For example, $4a + 3 + 2a = 6a + 3$ is a true statement because both expressions represent the same quantity. In this situation, the Commutative and Associative Properties of Addition make the combining of the like terms $4a + 2a$ equivalent to $6a$. The number 3 remains a constant in both expressions.



5 + 5 + 5 + 5 + 3 + 5 + 5 = 33 = 5 + 5 + 5 + 5 + 5 + 5 + 3 = 33

Substitution may also be used to determine if an equation is true. If $a = 5$, the sum of each expression is 33, making them equivalent.
(Othe equations can be true such as 1/2 = 0.5.)

An inequality is true when it states a correct relationship. For example, $4a + 3 \neq 6a + 3$ is a true statement because both expressions represent different quantities.



5 + 5 + 5 + 5 + 3 = 23 ≠ 5 + 5 + 5 + 5 + 5 + 5 + 3 = 33

Substitution may also be used to determine if an equation is true. If a=5, the sum of one expression is 23 and the other 33.
(Other inequalities can be true such as 4 > 2)

How is solving an equation or inequality a process of answering a question?

In essence, solving equations or inequalities is like asking, "For what numbers will this equation or inequality be true?"

INEQUALITY EXAMPLE: Shipping is free for all orders $24.99 and over at clothesfortrolls.com. What amounts can receive free shipping? An infinite number of values can receive free shipping--anything $24.99 and up. Let "*a*" represent free shipping orders, so a > $24.99.

EQUATION EXAMPLE: The librarian is filling a bookshelf designed for an E-Z Reader series. It can hold 125 books and she has placed 47 books on the shelf already. How many books can she add to the shelf to fill it? ($47 + a = 125$)

Properties of Equality

Reflexive	$a = a$ *a* is equivalent to *a*.	Subtraction	If $a = b$, then $a - c = b - c$ If I subtract the same amount from both sides, the equation is still equal.
Symmetric	If $a = b$, then $b = a$ *a* and *b* are equivalent.	Multiplication	If $a = b$, then $ac = bc$ If I multiply both sides by the same amount, the equation is still equal.
Transitive	If $a = b$ and $b = c$, then $a = c$ If *a* and *b* are equivalent and *b is equivalent to c*, then *a* is also equivalent to *c*.	Division	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$ If *a* and *b* are equal, then *a*/*c* is equal to *b*/*c* as long as *c* does not equal 0.
Addition	If $a = b$, then $a + c = b + c$ If I add the same amount to both sides, the equation is still equal.	Substitution	If $a = b$, then *b* may be substituted for *a* in any expression containing *a*. If *a* =15/7 and *b*=2 1/7, I can use either term to solve a problem because they are equivalent.

It is not expected that 6th grade students memorize the Properties of Equality or Inequality. However, much of their reasoning might be grounded in an innate sense of these properties. Being aware of them strengthens a teacher's ability to support the learning trajectories of students.

Properties of Inequality

There are three types of inequalities possible relating a to b: $a < b$, $a = b$, or $a > b$.

If $a > b$ and $b > c$ then $a > c$. If a is greater than b and b is greater than c, then a has to be greater than c also.	If $a > b$ and $c > 0$, then $ac > bc$ If I multiply a and b by the same whole number, then ac will be greater than bc.
If $a > b$, then $b < a$. If a is greater than b then b must be less than a.	If $a > b$ and $c < 0$, then $ac < bc$ If I multiply a and b by the same negative integer, than ac will be less than bc.
If $a > b$, then $-a < -b$ If a is greater than b, then the opposite of a is less than the opposite of b.	If $a > b$ and $c > 0$, then $a \div c > b \div c$ If a is greater than b and I divide both by the same positive integer, then a will still be greater than b.
If $a > b$, then $a \pm c > b \pm c$. If a is greater than b and I add or subtract the same amount to/from each quantity, a will still be greater than b.	If $a > b$ and $c < 0$, then $a \div c < b \div c$ If a is greater than b and I divide both by the same negative integer, then a will be less than b.

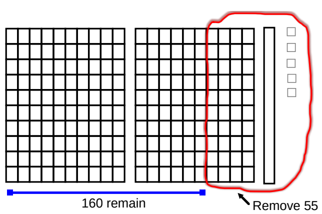
Writing and Solving Equations

$x + p = q$

Melrose Elementary 6th graders made $215 dollars on their school play. They made $55.00 on popcorn sales. How much did they earn from ticket sales?

Solution Methods

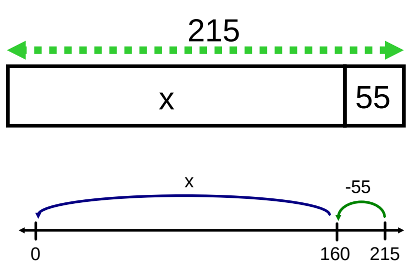
SET MODEL



ALGORITHMIC SOLUTION

1) Write equation. $x + 55 = 215$
2) Subtract 55 from each side. $x + 55 - 55 = 215 - 55$
3) Simplify. $x + 0 = 160$
4) Solve for x. $x = 160$

MEASUREMENT MODEL



NOTE

In $x + p = q$ and $px = q$:

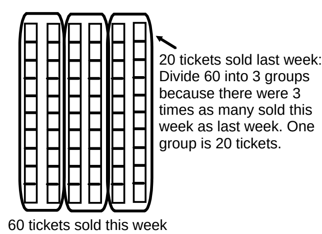
- **x is the variable, and**
- **p and q are constants.**

$px = q$

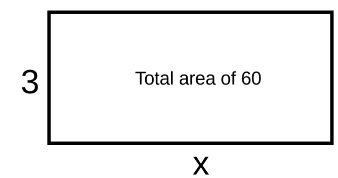
Melrose Elementary 6th graders sold three times as many tickets to the school play this week as they did last week. They sold 60 tickets this week. How many tickets did they sell the week before?

Solution Methods

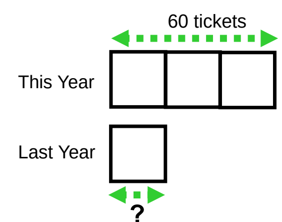
SET MODEL



ARRAY OR AREA MODEL

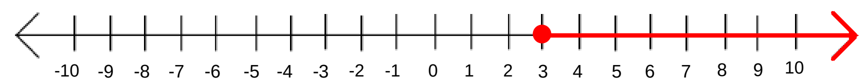
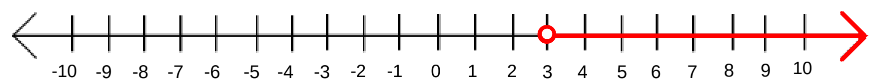
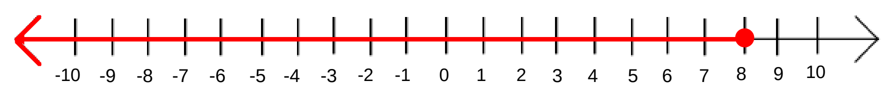


MEASUREMENT MODEL



ALGORITHMIC SOLUTION

1) Write equation. $3x = 60$
2) Divide both sides by 3. $\frac{3x}{3} = \frac{60}{3}$
3) Solve for x. $x = 20$

Inequalities in the form of *x* > *c* or *x* < *c*		
What are constraints or conditions in real-world situations or mathematical problems? Constraints or conditions limit the numerical solutions that can be given based on real-world situations or the dynamics of the math problem. It is important to remember the density of the number line to fully fathom all of the possible solutions. For example, if $x > 3$ there are an infinite number of whole number solutions possible. There are also an infinite number of fraction and decimal solutions. Any number greater than 3 is part of the solution set, so 3.00098, 3,098,287, and 3 ½ are all possible solutions.		
Modeling Solutions on a Number Line		
$x \geq 3$	Children 3 years old and older may ride any of the rides in the local amusement park. ***NOTE: The circle is filled indicating that 3 is part of the set.***	
$x > 3$	Children older than 3 may ride any of the rides in the local amusement part. ***NOTE: The circle is not filled indicating that 3 is not part of the set.***	
$x \leq 8$	Temperatures at Lake Chilly are expected to be 8° C and below on Tuesday. ***NOTE: The circle is filled indicating that 8 is part of the set.***	
$x < 8$	Temperatures at Lake Chilly are expected to be below 8°C on Tuesday. ***NOTE: The circle is not filled indicating that 8 is not part of the set.***