Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another, write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs, and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Connections to other grade levels

In K-3, students study arithmetic sequences such as skip counting. In Grades 4 and 5, students generate and analyze patterns that follow a given rule. They also study the relationships between corresponding terms.

What are quantitative relationships between dependent and independent variables?

Students should have opportunities to represent mathematical quantities in real-world situations so they can build understanding of meaning and use of variables. They can build quantitative reasoning by looking at the relationship between two variables that are related in such a way that one is affected by the other. The dependent variable, the output number, is affected by the independent variable, the input number.

How can quantitative relationships be represented?

Quantitative relationships can be represented in multiple ways: 1) through real-world context; 2) using a table of values; 3) creating a visual graph; and 4) writing symbolic equations. Each representation is a different way to communicate the same relationship. It is important that students conceptually recognize and develop sound understanding of the connections among these equivalent various mathematical representations.

Representing Quantitative Relationships

Real World Context:

A school is having a walk-a-thon for a fund raiser. Each student in the walk-a-thon must find sponsors to pledge $2.00 for each mile the student walks. Sponsors want to know how much money they would owe given the total distance the students would walk.

Table of Values:

A table of values provides an organized way to help students determine the quantitative relationships within the context.

Number of Miles Walked (Independent Variable known as *x*)	0	1	2	3	4	5	6
Total Dollars Sponsor Would Owe (Dependent Variable knows as *y*)	0	2	4	6	8	10	12

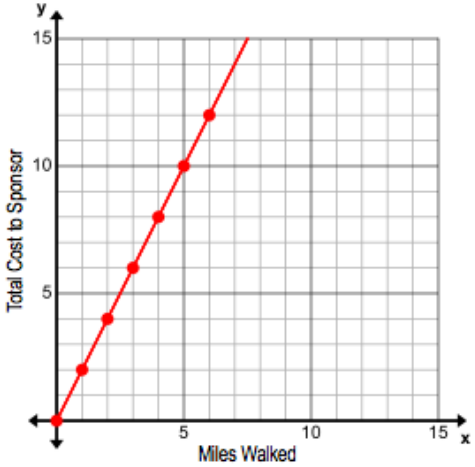
The first row in the table of values shows the number of miles the student walks, which is the input. The second row shows the total number of dollars the sponsor owes, which is the output. The quantitative relationship is evidenced in that the total amount the sponsor owes depends on how many miles the student walks. The table helps students recognize the pattern in the function. In this situation, the number of miles (*x*) is multiplied by the cost per miles ($2) to determine the total cost (*y*). By substituting any number of miles for *x*, students can find the total cost the sponsor owes.

A table of values can also be shown in a vertical format.

Miles	Dollars
0	0
1	2
2	4
3	6
4	8
5	10
6	12

Graphs:

Students can graph the quantitative relationship on a coordinate plane. When graphing the data, the horizontal *x*-axis represents the independent variable of miles walked, and the vertical *y*-axis represents the dependent variable of total dollars the sponsor owes. The graph gives the students a visual image that helps them describe the relationship between miles walked and total money owed. When connected, the points form a straight line, which means it is a linear function. The rate of change is constant meaning that for every mile walked, there is a two-dollar cost for the sponsor.



When representing quantitative relationships on a graph it is important to discuss whether the plotted points should or should not be connected. When graphing things that cannot be broken into smaller parts, like number of cars and riders per car, the points should not be connected. When graphing things that can be broken into smaller parts, like miles walked and dollars owed, the points should be connected. In other words, if it is reasonable within the context to have a value at any point on the line, the points should be connected. If it is not reasonable within the context to have a value at any point on the line, the points should not be connected.

Symbolic Equation:

The students can translate the verbal statement to develop an equation that represents the quantitative relationship of the context.

The total sponsor cost equals miles walked times $2.00.
or
dollars = $2.00 x miles
or
$d = 2m$