Geometry (Spatial Sense & Reasoning)

Geometry is the only CCSSM Domain that consistently appears in every grade level K-12. At all grade levels, geometry content goals can be summarized into four main geometric ideas:

1. **Shapes, Solids and Properties**: analyzes the attributes & properties of shapes in two and three dimensions
2. **Transformations:** examines translations, reflections, rotations (slides, flips, and turns) and the study of symmetry and the concept of similarity
3. **Visualization:** includes the recognition of shapes in the environment, identifying and developing relationships between two- and three-dimensional objects, and the ability to draw and recognize objects from different perspectives
4. **Location:** specifically determines the orientation of objects located on a plane or in space ~ often called 'coordinate geometry'

The CCSSM Geometry domain includes all four categories. Some CCSSM Geometry standards require the use of skills and knowledge from multiple categories. For example, Standard 1 requires students to find area of triangles, special quadrilaterals and other polygons by composing and decomposing. This requires students to assimilate their knowledge of shapes and properties and rectangular area with skills of geometric transformations and visualization. Standards 2 and 4 require students to use visualization as they work with two and three-dimensional shapes for a variety of purposes.

CCSSM Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world mathematical problems.

Solve real-world and mathematical problems involving area, surface area, and volume.

Standard 1

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles and decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.*

*Special quadrilaterals are trapezoids, parallelograms, kites, rectangles, rhombuses, and squares.

Connections to Prior Core Curriculum Standards In the Common Core State Standards for Mathematics, students begin to develop a conceptual understanding of **area of rectangles** in third grade. By the end of fourth grade, students should be able to apply the **area formula for rectangles** in real world and mathematical problems. In sixth grade, students **use their understanding of rectangular area to help them develop understanding of area of triangles, special quadrilaterals, and polygons**.

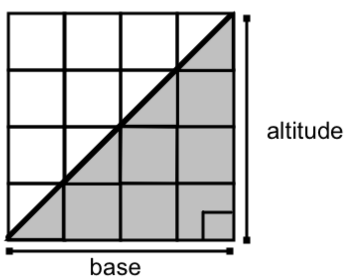
Useful Vocabulary It is beneficial to discuss the conventions of vocabulary as students transition from the formula for a rectangle to the formula for a triangle; the rectangle formula uses *length* and *width* while the triangle formula uses *base* and *height* to describe the same attributes. Students should learn/know and use ***compose, decompose, perpendicular, base***, and ***altitude*** (or ***height***). Any side of the triangle can be the ***base***. The ***altitude*** is the line segment drawn from a vertex, perpendicular to the line containing the opposite side. (The **height** is the actual distance; these terms are often interchangeable at this stage.) The right angle symbol is used to show perpendicularity. Other useful vocabulary are geometric transformations ~ ***flip (reflect), slide (translate)***, and ***turn (rotate).***

Cautionary note about formulas: Formulas can be a *tool* or *strategy* for geometric reasoning. The CCSSM does not explicitly cite area formulas in this standard as it does in other standards. However, reasoning about the area of triangles and special quadrilaterals requires students to have solid understanding of two area properties:

1) The area of a rectangle is *length x width* (area can be figured by calculating a number)
2) Congruent figures have the same area (area can be figured using visualization, shapes and properties, and transformations)

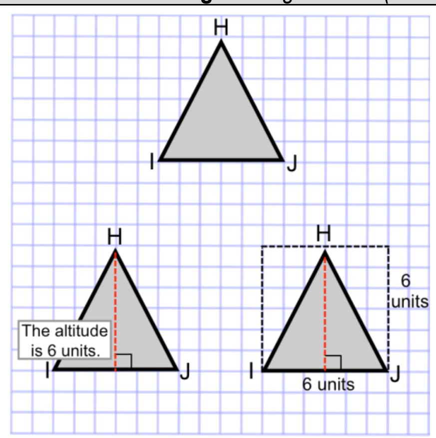
Finding the area of right triangles and other triangles by *composing* into rectangles. In this approach students use what they know about area of rectangles to determine area of triangles by composing them into rectangles. **Stage 1** is straight-forward and should not take much time for students to master the idea. During **Stage 2,** it is important for students to have multiple experiences using a variety of acute triangles. This will help them with obtuse triangles in **Stage 3**. In **Stage 4**, students can apply their understanding of area of triangles and rectangles to compose other polygons.

Stage 1 *Right Isosceles Triangles*



This is a good entry point for students to begin developing ideas about area of triangles. It is clear to see, to count, and to prove that the shaded area is half of the rectangle, $A = ½(b \times h)$. However, at this stage, it is unproductive for students to think of area as 'counting'. (In fact, this strategy only works for right isosceles triangles; it becomes increasingly difficult for other triangles.

Stage 2 *Height Inside (Acute Triangles)*

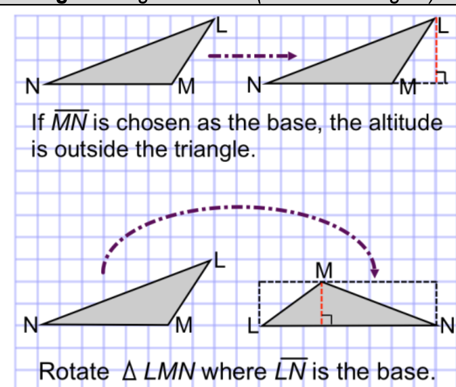


$\triangle HIJ$ can be decomposed into two right triangles by drawing the altitude and determining the height. Each triangle has a congruent 'match' that can be visualized by drawing the other part. From here, there are two entry points to finding the area of $\triangle HIJ$:

1) Find the area of each *right* triangle, then combine them, OR
2) Find the area of the entire rectangle and calculate half $A = ½(b \times h)$.

*** Use this strategy with a variety of acute triangles (isosceles, equilateral, and right). Experiment rotating triangles and using different sides as base.*

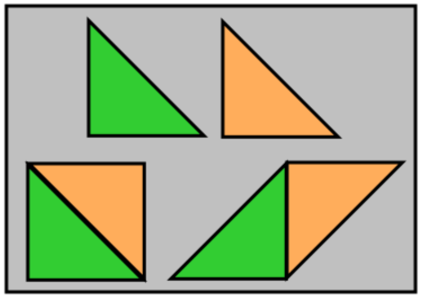
Stage 3 *Height Outside (Obtuse Triangles)*



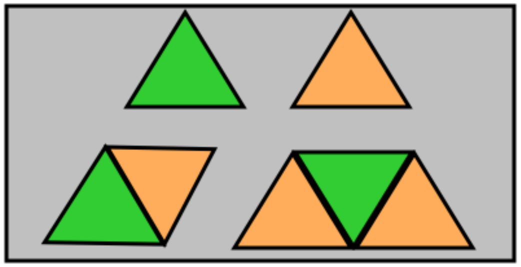
Obtuse triangles are more challenging and should be approached after students have good understanding of right and acute triangles. For many students, it is easier to select the longest side to be the base; then, the height is on the inside, and it allows them to use the same strategy as Stage 2.

Stage 4 *Composing Triangles into Special Quadrilaterals & Other Polygons ~ What kind of quadrilaterals can be made using congruent triangles?*
In this stage, students use a variety of triangles to compose special quadrilaterals and other polygons. Some examples are shown below.

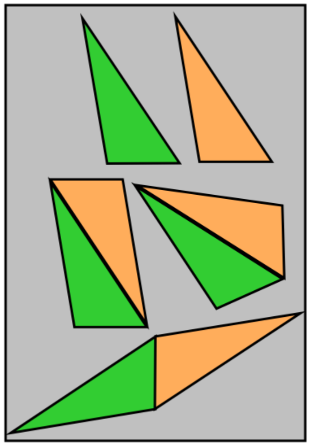
Congruent right triangles can make…



Congruent isosceles triangles can make…



Congruent obtuse triangles can make…

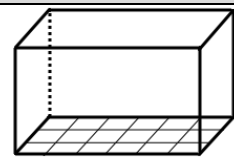


Finding the area of special quadrilaterals and other polygons by decomposing into triangles and other shapes. *This sequence is a reversal of the first* (using triangles to compose quadrilaterals). The focus is on decomposing parallelograms into congruent triangles. Using congruent triangles, it can be determined that the area of a square, parallelogram, or rhombus is the same as the area of a rectangle (*A = b x h*). However, this idea does not hold true for trapezoids. Consequently, trapezoids will be decomposed into triangles and other shapes, and the formula is not the same. The formula for a kite is also different. While kites can be decomposed into two congruent triangles, they are not parallelograms.

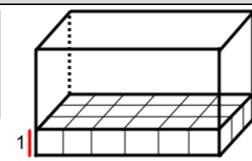
Geometry (Spatial Sense & Reasoning)
Solve real-world and mathematical problems involving area, surface area, and volume.
Standard 2
Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths (e.g., 3 ½ x 2 x 6), and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l • w • h and V = b • h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.*
**Clarification: It is not intended that this be modeled physically; it should be a conceptual activity modeled with drawings and diagrams.*

Connections to Prior Core Curriculum Standards
In the Common Core State Standards for Mathematics, students begin to develop a conceptual understanding of liquid volume (e.g., cups, liters) in third grade and use four operations to solve problems involving ***liquid volume*** in fourth grade. In fifth grade, students begin conceptual understanding of ***volume of solid figures using cubic units*** and move toward using the volume formula to solve problems involving right rectangular prisms. In sixth grade, students **extend their computational skills** (*V = l • w • h and V = b • h)* to include ***right rectangular prisms with fractional edge lengths***.

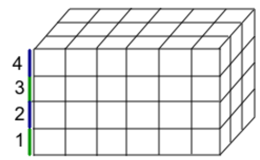
V = b • h
This formula illustrates the 'layering' approach to figuring volume.



Base is 3 x 6.
Area of base is
18 **square** units.

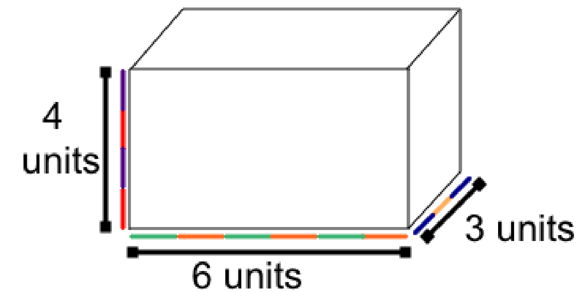


Base holds 1
set of 18
cubic units.



The layer of 18 **cubes**
fills the box 4 times.
V = 18 x 4 **cubic** units

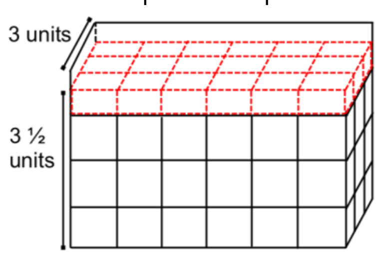
V = l • w • h
This formula is more abstract; it is more difficult to connect to the idea of 'filling'.



V = 6 x 3 x 4

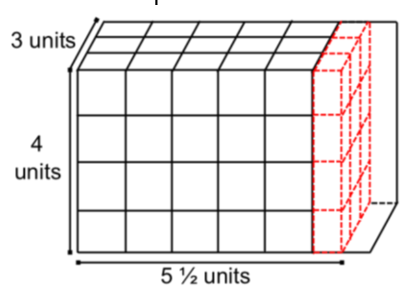
In sixth grade, students use these same strategies and formulas to determine the volume of right rectangular prisms with fractional edge lengths. It is helpful to scaffold student learning by beginning with only one fractional edge length **and** by using benchmark unit fractions (e.g., ¼, ½). Students can begin to visualize a container that can *almost* be filled with complete layers. The gap can occur at the top (e.g., filling a box), on the side (e.g., filling a shelf), or the front/back (e.g., filling a filing cabinet). Limiting fractions to benchmark unit fractions allows students opportunity to convert benchmark fractions to decimals for easier computation.

Gap on the Top



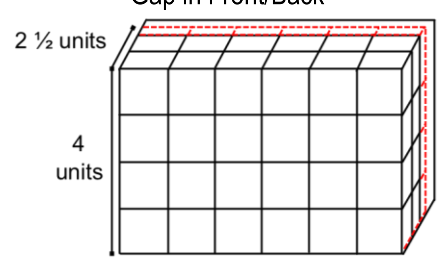
3 whole layers, then ½ of another layer

Gap on the Side



5 whole layers, then ½ of another layer

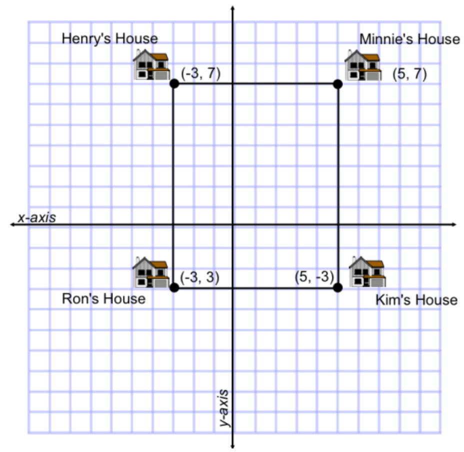
Gap in Front/Back



2 whole layers, then ½ of another layer

Geometry (Spatial Sense & Reasoning)
Solve real-world and mathematical problems involving area, surface area, and volume.
Standard 3
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

The distance between two points may be determined by counting units. As students work, encourage them to use what they know about coordinates to find distances without 'counting' on the grid.
(Opportunity for connection to 6.EE.)



Example
Four friends used GoogleMap to map out their neighborhood. They discovered that their houses form a rectangle. Use a coordinate grid to plot their houses; then answer the questions.

Henry's House (-3, 7) Ron's House (-3, 3)
Minnie's House (5, 7) Kim's House (5, -3)

How many units away is Henry's house from Ron's house?
How many units away is Minnie's house from Henry's house?
Who lives closer to Kim – Ron or Minnie? How do you know?

Geometry (Spatial Sense & Reasoning)
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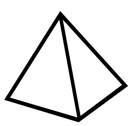
Standard 4
Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world mathematical problems.

A ***geometry net*** is a 2-dimensional representation of a 3-dimensional figure. Geometry nets can be folded to create the figure. Nets provide a visualization tool that help students…

- ***see the relationships*** between two- and three-dimensional figures, the 'folding' and 'unfolding'. http://www.mathsnet.net/geometry/solid/nets.html
- ***examine the properties*** of 3-dimensional figures. For example, the difference between a triangular pyramid (tetrahedron) and a rectangular pyramid is shown by comparing and contrasting each pyramid's net. By 'unfolding' each figure, students can more clearly see the rectangles and triangles that make-up the pyramids. They both have a pyramid 'shape', but the triangular pyramid has a triangular base with three triangular faces. The rectangular pyramid has a square base with four triangular faces.
- ***use formulas and other strategies to find the surface area of 3-dimensional figures.*** *(The CCSSM Grade 6 integrates Geometry surface area formulas in 6.EE.)*

Students develop the ability to move from 2-dimensional to 3-dimensional thinking at varied rates. Students need multiple exposures and opportunities to 'fold' and 'unfold' dimensions.

Example: *Is it a tetrahedron or a pyramid?*



Without the dotted lines that show the third dimension (depth), it would be very difficult to determine the details of this solid. When 'unfolded', the shape of each face is distinctly observable.