Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for working with unit rates in this grade are limited to non-complex fractions.)
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    a. Make *tables of equivalent ratios* relating quantities with whole number measurements, find missing values in the tables, and *plot the pairs of values* on the coordinate plane. Use tables to compare ratios.
    b. Solve *unit rate problems* including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
    c. *Find a percent of a quantity as a rate per 100* (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
    d. Use *ratio reasoning to convert measurement units*; *manipulate and transform units appropriately* when multiplying or dividing quantities.

Connections to other grade levels

A formal study of ratio and proportional relationships is only provided in grades 6 and 7. In 6th grade, students develop the concept of ratio and rate reasoning. In 7th grade, students focus heavily on proportions and proportional reasoning.

What is ratio and ratio reasoning?

Ratios are not numbers in the typical sense. They cannot be counted or placed on a number line. They are a way of thinking and talking about relationships between quantities.

- Students are frequently exposed to equivalent ratios in multiplication tables. For example 1/3 is often stated as equivalent to 3/9, which is a true statement. This relationship of equivalence can be very challenging for students to understand because it appears that they are not numerically the same. However, from a ratio perspective 1 to 3 has the same relationship as 3 to 9. In this way, students are thinking about a ratio relationship between two quantities.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	**1**	2	**3**	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	**3**	6	**9**	12	15	18	21	24

- Ratio reasoning involves attending to covariation. This means that students must hang onto the idea that one quantity changes or varies in relation to another quantity. For example, 1 cup of sugar is used for every 3 cups of flour in a recipe. **IF** 2 cups of sugar are used, **THEN** the flour must change or vary in the same way. (**IF--THEN statements might help children process the idea of relationship between quantities.**) In this case, the amount of sugar doubled, so the amount of flour should also double. Students must hold onto the idea that a change in one quantity creates a need for change in the other quantity. While this reasoning is fairly intuitive for adults, it is not always easy for children to grasp. Many opportunities to reason about ratio helps them develop the ability to attend to covariation.

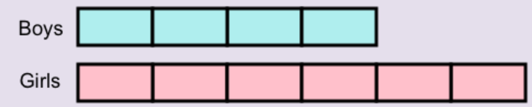
Three Types of Ratios

SAMPLE PROBLEM STEM:

There are 10 students lined up to play dodge ball. 4 of them are boys and 6 of them are girls.

1 - Part-to-Part Ratios are ratios that relate one part of a whole to another part of a whole. In this situation, there are 4 boys for every 6 girls, typically represented as 4:6.

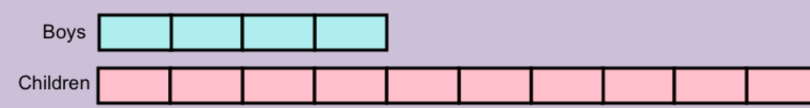
What is the ratio of boys to girls in the line?



Notice that the units being compared are from the same group: the children in line. Also, more than two quantities can be compared.
EXTENDING THINKING: Perhaps several parents were in the line, so the ratio could be 4 girls to 5 boys to 1 parent (4:5:1).

2 - Part-to-Whole Ratios are ratios that relate one part of a whole to the whole. In this situation, there are 4 boys for every 10 children, which can be written as 4:10. On the other hand, 6 girls for every 10 children can be written as 6:10.

What is the ratio of boys to children in line?



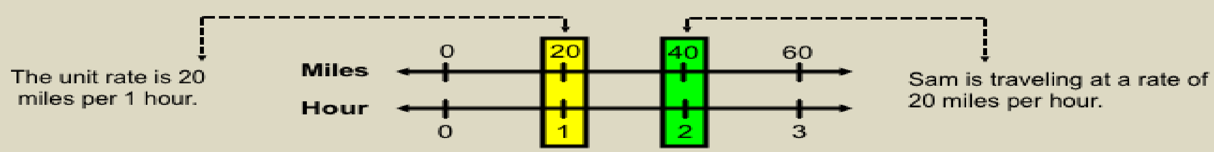
Notice that the units being compared are from the same group: the children in line.

FRACTIONS and PART-TO-WHOLE RATIOS both represent a comparison of parts to wholes. This is the overlapping area when fractions are also ratios. Fractions are NOT ratios in terms of part-to-part or rate comparisons.
PERCENTAGES can be thought of as **PART-TO-WHOLE RATIOS** because 100 is the unit whole around which quantities are being compared.

SAMPLE PROBLEM STEM:

Sam bikes 20 miles in 1 hour. If he continues biking at this rate…..

3 - Rates as Ratios can be thought of in several ways. For this discussion, we'll think of rates as ratios that relate different units. In this example, distance is being related or compared to time. Common rates include miles compared to hours (mph), miles compared to gallons (mpg), wages per hour, or price per pound.



Notice that the units being compared are different: miles and hours.

A **UNIT RATE is when the input unit is 1** . Students encounter unit rates more often than other rates in their everyday experiences. For example, most items at the grocery store are marked using unit rate pricing (bananas are 59 cents per one pound or spaghetti noodles are $1.50 per single package). In the example above, the unit rate is 20 miles for every 1 hour.
PERCENTAGES can also be thought of as **RATES** per 100. For example, a score of 89% represents the part (89%) out of the whole (100%). This holds true even when the part is greater than 100%. A score including extra credit points could be 102% out of 100%. In other words, the part being considered comprises the entire whole plus 2% more.

> **Point of Confusion for Students**: In our language structure, we tend to talk about unit rates by saying the output and then the input (miles per hour). This might feel backwards for students when they work with tables or graphs since they are typically oriented from input to output (hours per mile).

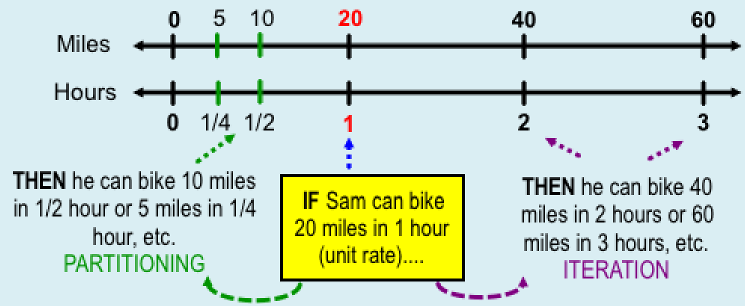
Representing Ratios

Representing Ratios

While ratios can be represented in a multitude of ways, this section provides examples of representations based on the language of the core.

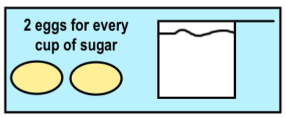
Two ideas to think about when representing ratios are iteration and partitioning. ***Iterating*** (repeating a unit) is done by making multiple copies of the unit. ***Partitioning*** (splitting a unit) is done by subdividing a unit. Both of these methods expose the MULTIPLICATIVE nature of ratios.

Partitioning or Iterating a unit

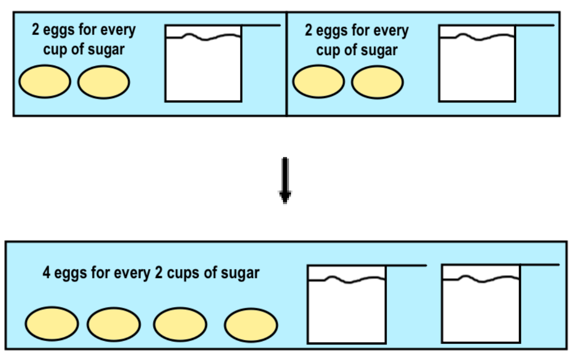


The relationship between quantities within the ratio does not change with partitioning or iterating. In other words, the ratio stays the same.

Concrete Models: Students must often draw concrete examples before they are able to use other methods for representing ratios. For example, 2 eggs for every 1 cup of sugar in a recipe might look like the following.

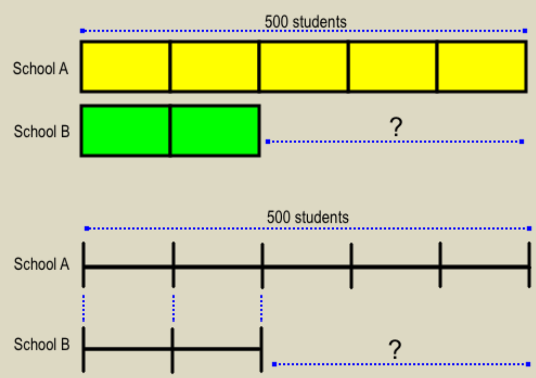


Students might create an equivalent ratio by iterating the unit like this and then combining the appropriate quantities to make the ratio more evident.



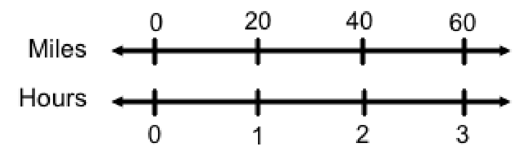
Tape Diagram: According to the CCSSM glossary, *a tape diagram is a drawing that looks like a segment of tape, used to illustrate number relationships. It is also known as a strip diagram, bar model, fraction strip, or length model.* These models work particularly well to show comparisons between part-to-part or part-to-whole ratios.

COMPARISON (part-part) MODEL: *School A has 500 students, which is 2 ½ (which is equal to 5/2) times as many students as School B. How many more students attend School A than School B?*

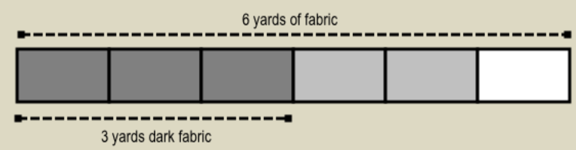


Double Number Line: The idea behind a double line is to be able to graphically represent two quantities simultaneously. This model works well to show ratios as rates.

Double Line Method



COMPARISON (part-whole) MODEL: *Sue needed 6 yards of fabric to make a quilt. She needed to purchase 3 parts dark fabric: 2 parts medium fabric: 1 part light fabric. How much dark fabric should she purchase?*



Extend: *How much dark fabric will she need to purchase for a quilt requiring 12 yards?*

Representing Ratios *continued*

Table of Equivalent Ratio: A table of ratios allows students to organize information efficiently, isolate coordinate pairs for graphing, and to generate equations.

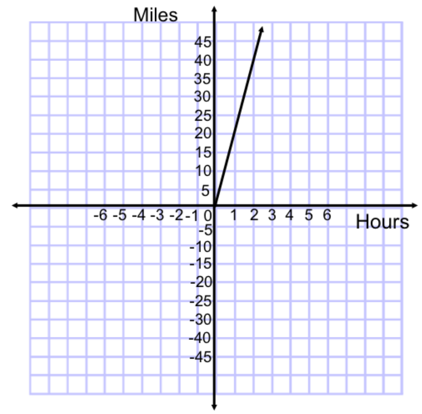
> ***If Sam can bike 40 miles in 2 hours, how many miles can he bike in 3 hours? What is his rate per hour?***

Hours	Miles
0	0
1	20
2	40
3	60

The "hours" column represents the input or *x*. The "miles" column represents the output or *y*. Tables can also be constructed horizontally with *x* generally on the top row.

The **UNIT RATE** is when the input is 1. (Sam can travel 20 miles in 1 hour.)

Graphing Equivalent Ratios: Equivalent ratios create a straight line passing through the origin. Helping students make this connection will support the work they'll be doing during the Expressions and Equations Domain, Standard 7.



The **UNIT RATE** is 1 on the x-axis.

Equations: The equations generated during the ratio unit will be unique in that they follow the form of y = mx or px = q (these notations represent the same thing). The intercept (location the line crosses on the y-axis) will always be zero. Keep in mind that the writing of equations is part of the Ratio AND the Expressions & Equation Clusters.

Miles traveled = 20 • number of hours

or

$y = 20x$

The **UNIT RATE** is when the input is 1 (Sam can travel 20 miles in 1 hour.)

Ratio Language, Symbols, and Real-World Connections: Students are exposed to ratio language every day, and it comes in many forms. For example, *1 can of frozen concentrate is mixed with 3 cans of water to make juice*. A few examples of symbolic notation and ratio language for this ratio are listed below.

Ratio of Concentrate to Water (Part-to-Part)		
1:3	1 to 3	1 can concentrate **per** 3 cans water

Ratio of Concentrate to Juice (Part-to-Whole)			
1:4	1 to 4	¼	1 can concentrate **per** 4 containers of juice

<u>Examples of Real-world Connections</u>

Utility Prices: cost per gallon, cost per kilowatt hour

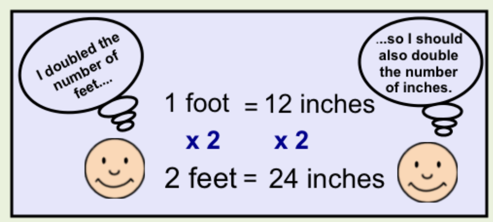
Communication: cost per text, cost per minute

Time: days per week, minutes per hour, etc.

Medical: heart beats per minute

Ratios and Measurement: Students can use ratio reasoning to think about the relationship between measurement units. This helps them make sense of conversion procedures because it adds a layer of reasoning that is often missing in traditional approaches. See the example below.

IF *1 foot is comprised of 12 inches,* ***THEN*** *2 feet will be comprised of HOW MANY inches?*

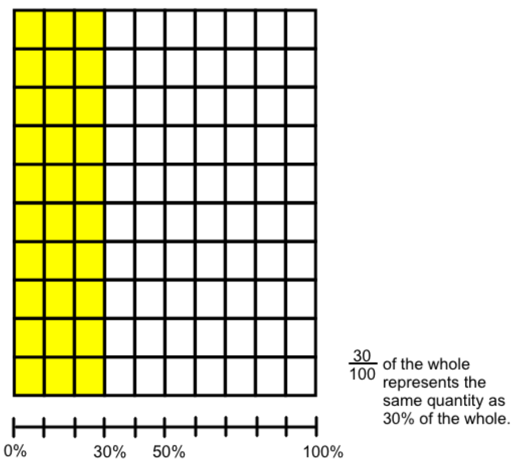


The complexity of the problem can increase while still maintaining the idea of ratio. (Think of it in terms of iterating or partitioning a unit.) A thought process might be, "If 1 foot is comprised of 12 inches, then 1 ½ feet must be 12 inches + 6 inches (half a foot), so 1 ½ feet contains 18 inches". Of course, this idea works with division. ½ foot is HOW MANY inches? If I cut a foot in half, then I also have to cut the number of inches in half, so there are 6 inches in ½ foot.

The idea behind this connection is that students begin to see the covariation between measurement units. As one unit changes, the other also changes in the same way.

PERCENTS AS RATIOS

Finding a Percent of a Quantity as a Rate per Hundred: Percents are a unique kind of rate because they represent a rate per 100. Students should understand that hundredths (base-ten fractions) and percentages are synonymous, though the symbolic notation is different. In other words, 30% is the same as 30/100.



Any given percent of a quantity can be written fractionally as x/100 times the quantity, or x/100 "of" the quantity. Consider 25% of $8.00 in terms of hundredths.

25% of $8.00 can be solved by writing the percent in hundredths and then multiplying by the quantity.

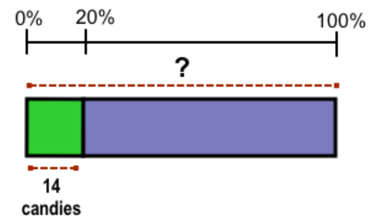
$$\frac{25}{100} \times 8 = 2$$

25% of $8.00 is $2.00

Analyzing the story problem to find the whole, part, and percent will help students think of the 3 terms in the situation.

Whole: $8.00
Part: Unknown
Percent: 25%

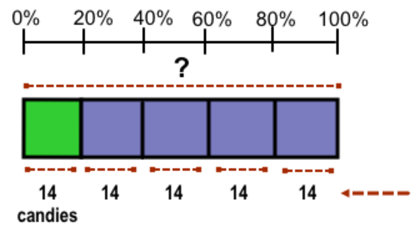
Finding the Whole Given a Part and a Percent: The tape diagram (bar model) provides the imagery needed to help students conceptualize the whole in terms of the part and percent. The example below takes the approach of capturing the precise information from the story problem in the bar model. (Scaffolding for some students might include dividing the number into 10 sections with 10% intervals if they're having a difficult time locating the percent.)



There are 14 candies in a bag that is 20% full. How many candies are in a full bag?

Whole: unknown (How many candies in a full bag?)
Part: 14 candies
Percent: 20%

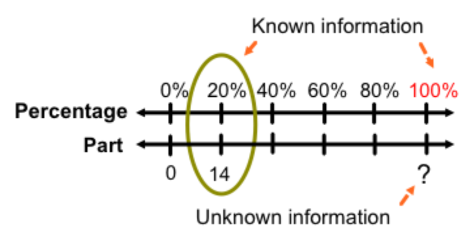
There are several strategies students can use to find the unknown whole after creating this diagram. For example, the bar can be subdivided into 20% sections, which provides a tool for adding up (or multiplying) to find the total quantity.



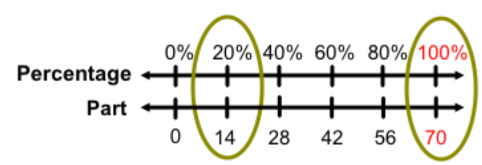
If there are 14 candies in 20%, then I can fill in that amount in the other 20% sections of the bar. Then I can add all the quantities: 14 + 14 + 14 + 14 + 14 = 70 or 14(5) = 70.

A **double bar diagram** also helps students solve the problem pictorially.

Step 1: Identify the information



Step 2: Fill in equivalent ratios to locate solution.



A **function or input/output table** can be used in a similar way.

Percentage	0%	20%	40%	60%	80%	100%
Part	0	14	28	42	56	70