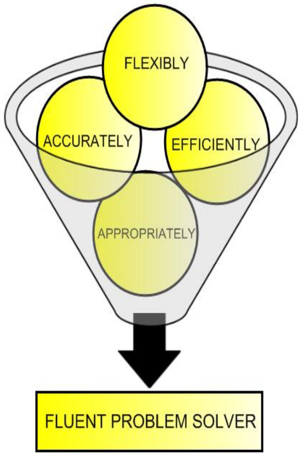
Compute fluently with multi-digit numbers and find common factors and multiples.
Standard 2: Fluently divide multi-digit numbers using the standard algorithm.
Standard 3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
Standard 4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).

Connections to Other Grade Levels: In previous grades, students construct understanding of addition, subtraction, and multiplication through concrete models and visual representations, applying that understanding to standard algorithms. Division is introduced through concrete and pictorial representations, and 6th grade students formalize that work with division computation using the standard algorithm. Students begin working with factors and multiples in 3rd grade and continue building fluency through 5th grade, which includes attention to prime and composite numbers. Special types of multiples such as Least Common Multiple and factors such as Greatest Common Factor are not introduced until 6th grade. In 7th grade, factoring linear expressions with rational coefficients is supported by elementary work with factors and multiples. Students will continue this thinking in more abstract terms throughout Secondary I. In Secondary 2, they will apply this thinking to the factoring of polynomials.



WHAT DOES IT MEAN TO COMPUTE FLUENTLY?
Procedural fluency is defined by the Common Core as, "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits to how long computation should take. *Memorization* is the rapid recall of arithmetic facts or mathematical procedures and is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experiences. Therefore, the building blocks for developing fluency occur through connecting concrete, pictorial, and abstract understanding or skills within any given Domain or Standard.

FLEXIBLY: A student who is mathematically flexible recognizes that numbers can be composed and decomposed. They recognize relationships between numbers and/or operations and use them as a tool for simplifying problems and can solve them quickly.

ACCURATELY: A student who is mathematically accurate is able to determine the correct answer to a task or problem whether it is contextualized or de-contextualized.

EFFICIENTLY: A student who is mathematically efficient is able to choose an effective computation or problem solving strategy including the standard algorithm given the constraints or conditions within the problem. Short cuts are understood and used deliberately.

APPROPRIATELY: A student who is mathematically appropriate is able to discern more than just a fast way to solve the problem, but is also able to recognize why and when a particular strategy or the standard algorithm is the best choice.

Students should leave sixth grade fluent in the use of standard algorithms when operating with whole and decimal numbers. That means that they engage in the process flexibly, efficiently, accurately, and appropriately. Ideally, this fluency is based on deep understanding of why the standard algorithms work developed in previous grade levels.

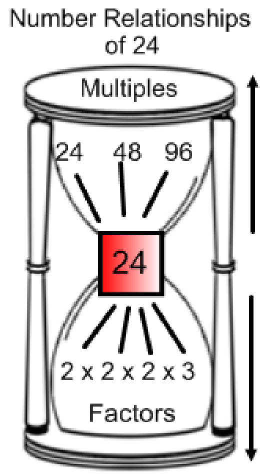
Least Common Multiple and Greatest Common Factor

The Fundamental Theorem of Arithmetic states that every number > 1 can be written as a unique product of primes.

Factors and Multiples are only associated with the operations of multiplication and division. In earlier grades, students compose and decompose numbers using addition and subtraction. In the upper grades, students compose and decompose numbers using multiplication and division. Factors and multiples play an important role in that work.

Why do students struggle with GCF and LCM?

- The Greatest Common Factor (GCF) and Least Common Multiple (LCM) are often taught as isolated topics. As a result, it is common for students to confuse the two especially if they lack an understanding of the relationship between factors and multiples. If students fail to see factors and multiples in a general sense, they will more likely struggle to understand specialized factors and multiples such as GCF and LCM.
- The language associated with GCF and LCM also creates some confusion for students. Finding the "greatest" seems like it should be a number larger than the ones being factored. Likewise, "least" seems like it should be a number smaller than the numbers being factored.



Helping Students Understand Factors and Multiples: Composition and Decomposition of Numbers

One way to build this understanding is to provide opportunities for students to think about number relationships. Specifically, numbers can be composed and decomposed in a multitude of ways. For example, the number 24 can be thought of as being composed of factors and it can also be thought as a starting point for creating multiples (of 24). Students should come to understand that all of the factors of 24 are also factors of all the multiples of 24 because those same "building blocks" for creating 24 remain within its multiples. Keep in mind, though, that additional factors are added in the multiples of 24. Additionally, developing fluency with factors and multiples encourages greater success with multiplication and division.

Knowledge of Factors and Multiples Support Work in Other Mathematical Areas

- Multiples and factors impact mathematics in many areas of mathematics. For example, knowledge of factors and multiples make fraction simplification easier.

$$\frac{18}{24} = \frac{2 \times 3 \times 3}{2 \times 2 \times 2 \times 3} \rightarrow \frac{18}{24} = \frac{\cancel{2} \times 3 \times \cancel{3}}{\cancel{2} \times 2 \times 2 \times \cancel{3}} \rightarrow \frac{3}{4}$$

- In addition, factors and multiples are also significant for solving many different types of story problems. Consider the story problem: *Mike runs every three days. Jose runs every four days. When will Jose and Mike run on the same days?* As you work with students, you will find that multiples and factors come up in many mathematical conversations. Strategies for helping with the mechanics of working with factors and multiples, along with the distributive property, are described below.

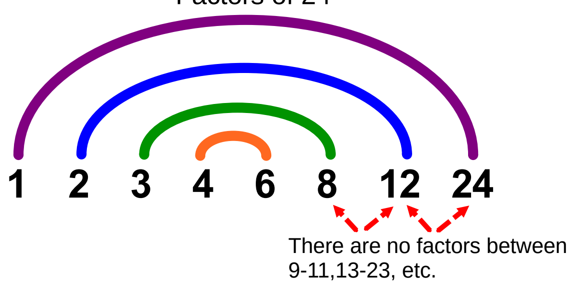
Greatest Common Factor & Distributive Property: Factor Rainbow Strategy

FACTOR RAINBOW: The factor rainbow assists students in seeing the range of numbers possible rather than randomly generating lists of numbers. Another benefit is that factors are arranged in order from least to greatest.

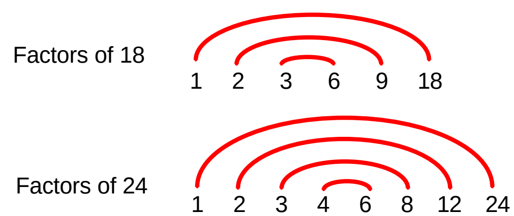
Example for finding the factors of 24: Start with 1 and 24, which is the range for finding possible factors and the greatest and smallest factors possible. Next, determine if the next prime number has a factor that creates 24. For example, 2 and 12 are factors of 24. This allows students to see that there are no possible factors between 13 and 23. Continue in this manner until all factors are determined.

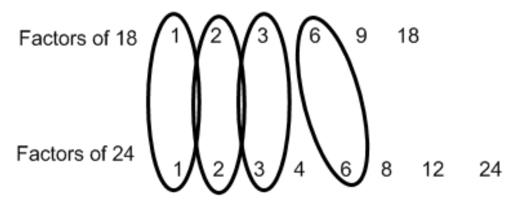
Creating a factor rainbow to find the factors of 24:

Factors of 24



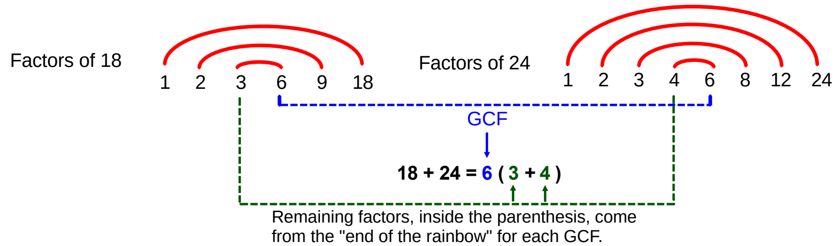
Finding the GCF of two numbers using a Factor Rainbow: Determine factors using the factor rainbow and then identify common factors. The GCF is the largest factor both numbers have in common. In this case, the Greatest Common Factor is 6.





Remember 1 is a factor for every number, but it is not a PRIME factor. The Factor Rainbow is effective for finding factors, but could create some confusion about prime factors. Once students are able to find factors, they should progress to the Factor Trees strategy, which does not add confusion between factors in general and prime factors.

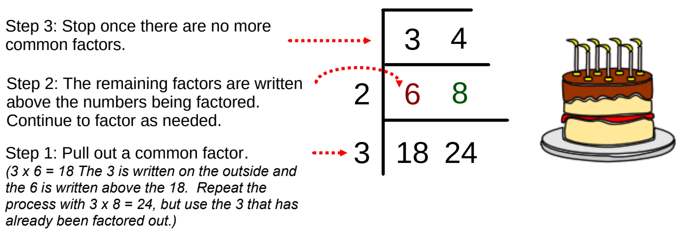
DISTRIBUTIVE PROERTY & FACTOR RAINBOW (Expressing the sum of two whole numbers with a common factor as a multiple of two whole numbers with no common factor.)



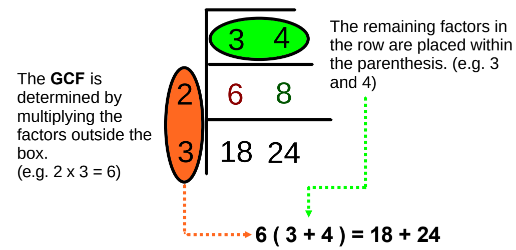
After determining the Greatest Common Factor using the Factor Rainbow, the remaining factors to be placed within the parenthesis can be located by finding the number at the "other end of the rainbow" from the GCF. For example, 6 is the GCF for 18 and 24. The factors at the other end of the rainbow are 3 (for factors of 18) and 4 (for factors of 24).

Greatest Common Factor & Distributive Property: Birthday Cake Strategy

This strategy provides a tidy structure for helping students determine the GCF and apply the Distributive Property to the factored expression 6 (3 + 4).



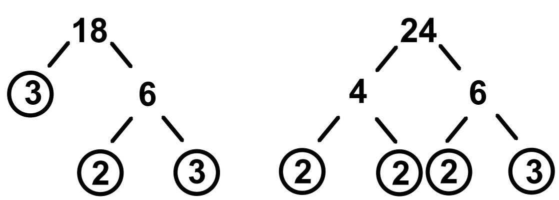
DISTRIBUTIVE PROPERTY & BIRTHDAY CAKE (Expressing the sum of two whole numbers with a common factor as a multiple of two whole numbers with no common factor)



The birthday cake strategy is effective, but it is not the common middle school approach. Therefore, the factor trees strategy is recommended for most students.

Greatest Common Factor & Distributive Property: Factor Trees Strategy (Recommended Strategy)

Begin with the traditional factor tree strategy for finding primes, and then prime factor both of the numbers being considered.



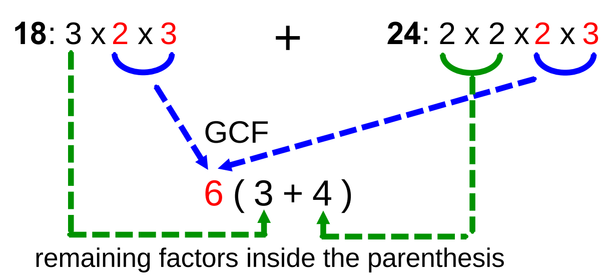
PRIME FACTORIZATION

18: 3 x 2 x 3

24: 2 x 2 x 2 x 3

DISTRIBUTIVE PROPERTY & FACTOR TREES (Expressing the sum of two whole numbers with a common factor as a multiple of two whole numbers with no common factor)

18 + 24 = 6 (3 + 4)

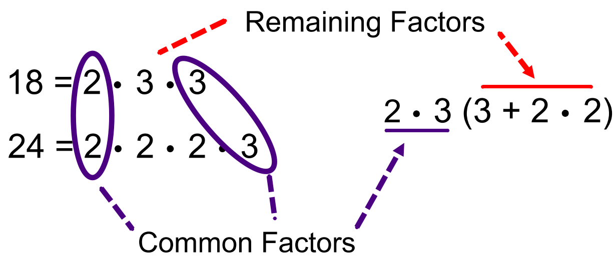


This image illustrates the relationship between "pulling out a factor" from two numbers and GCF in order to distribute it over addition. The Greatest Common Factor makes an excellent choice for this because it leaves the smallest numbers possible inside the parenthesis.

Greatest Common Factor & Least Common Multiple

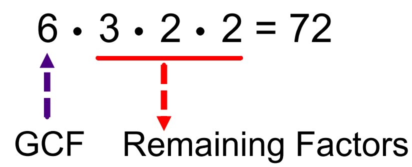
18 + 24

Greatest Common Factor



GCF is 2 ● 3 = 6

Least Common Multiple



LCM is 72

The GCF and LCM are important tools for helping students develop greater capacity for seeing and using number relationships to solve problems.