

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

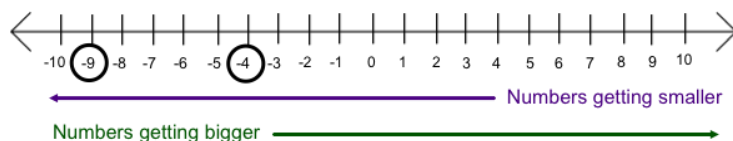
Connections to other grade levels

Integers: The study of integers begins in 6th grade. This introduction is focused on developing an understanding of the meaning of integers in real-world contexts. In 7th grade, students extend their understandings to explain and interpret rules for operating with negative integers by applying properties of operations.

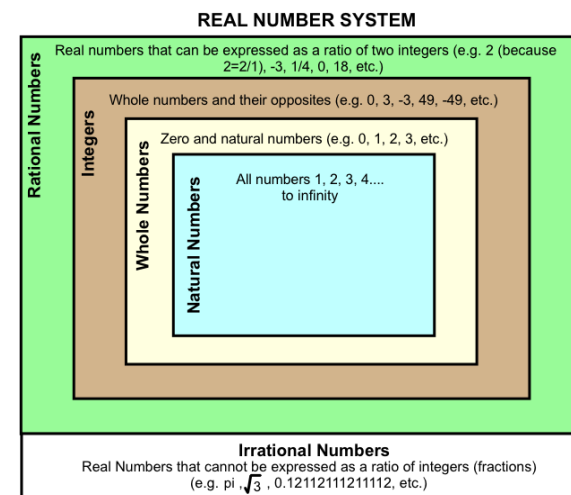
Coordinates: In 5th grade, students plot points in Quadrant 1 of the coordinate plane. 6th grade students build from this experience and plot coordinate pairs in all quadrants on the coordinate plane.

What makes integers difficult for students?

- **Whole Number Thinking vs. Integer Thinking:** Experience with whole numbers leads students to think that the 9 will always be greater than 4, regardless of the situation. As students encounter -9 and -4, they apply this background to the new situation and consider $-9 > -4$, which is not true. One way to remediate this misconception is to examine a number line with specific attention to its structure. Numbers to the right of another number are always greater and numbers to the left of another number are always less, regardless of the sign.



- **Lack of Context:** Integers are frequently presented with little real-world meaning. Contextualizing numbers accesses student schema and provides more opportunity for reasoning. For example, it is -9°F on a November day in Juno, Alaska at 8:00 a.m. By 10:00 a.m., it is -4°F . Have the temperatures dropped or risen? Is it colder at 10:00 a.m. or 8:00 a.m.? Engaging schema should lead students to conclude that generally speaking, temperatures rise during the morning so -4°F is actually warmer than -9°F , even though 9 is a larger number than 4 in whole numbers.
- **Notation of Integers:** It is common for students to think that -4 is the same or very similar to *subtract 4*. To further confuse things, many times we actually say, "It is minus four degrees." As student develop integer reasoning, using the term "negative" should help clear up some confusion.

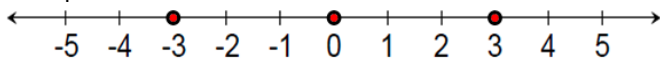


The colored section of this chart can be thought of as stacking on top of each other like Russian stacking dolls with natural numbers (sometimes called counting numbers) being the smallest "doll" or set of numbers. Whole numbers include all natural numbers, but also incorporate zero. Integers include all natural and whole numbers and their opposites. Finally, rational numbers contain all natural and whole numbers and their opposites along with fractions and decimals. They can be written in fractional form. Irrational numbers are also part of the Real Number System. However, they cannot be written as a fraction.

Representing Integers

Horizontal and Vertical Number Lines

Number lines are an effective tool for thinking about positive and negative integers and their relationship to zero. The use of horizontal and vertical numbers lines also supports student thinking on a coordinate plane.

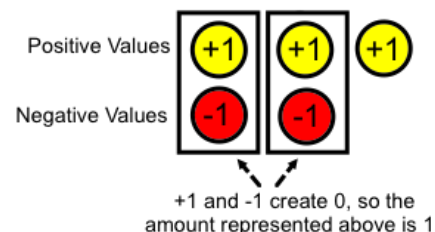


Key mathematical features for these standards:

- Rational Numbers are points on the number line.
- All numbers have an opposite on the number line. Opposites are the same distance from zero. 3 and -3 are both 3 “jumps” away from 0. Zero is its own opposite.
- The opposite of an opposite is the number itself. Symbolically, the opposite of the opposite is written $-(-3)$. This can be conceptualized by breaking it down into steps beginning with the quantity in parenthesis: take the opposite of (-3) , which is 3. Next, address the negative sign outside the parenthesis by taking the opposite of 3, which is -3. This illustrates the movement back to the original quantity, which is at the heart of this standard. This also illustrates that the negative sign is sometimes used synonymously with the term “opposite of”.

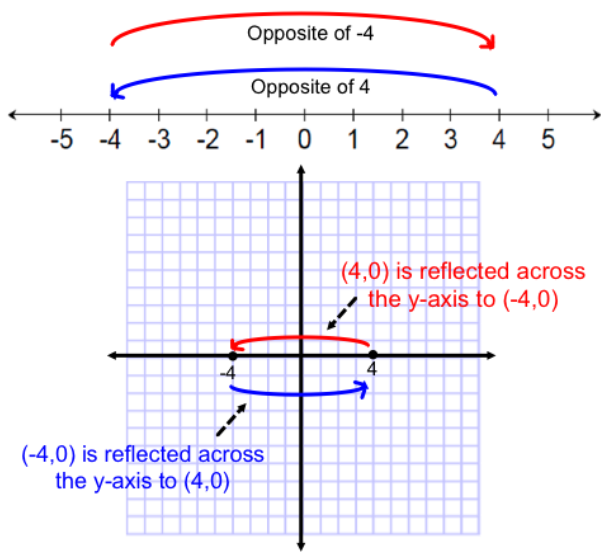
Charge Model

The charge model assigns a negative or positive value to two different objects within a set. It is often seen as red and yellow counters or two different colors of cubes. The actual object being used is not as important as the distinction that one item represents negative quantities and the other positive quantities. This is a fairly abstract concept and some students struggle with assigning a representation to a negative value. However, it helps them see zero in a new and necessary way.

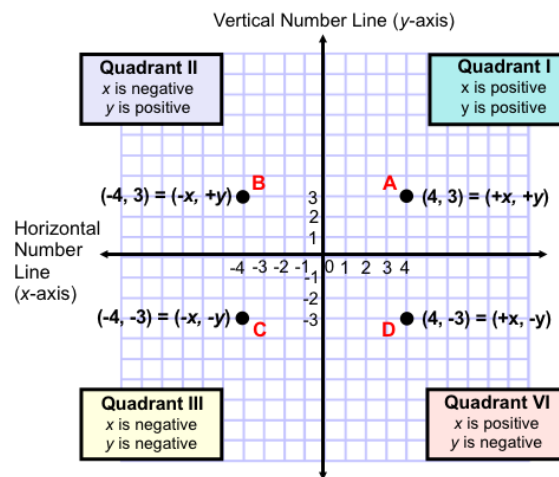


Number Lines and the Coordinate Plane

Taking the opposite of a rational number can be viewed as a reflection across the x or y axis on a coordinate plane.



When two ordered pairs differ only by signs, the location of the points are related by reflections across one or both axes. Ample practice should help students both see and trust the patterns of shared numbers and/or signs. As students begin to see the interplay between coordinates, they are preparing for future work with lines and functions.



Reflections Across One Axes

~Point D is a reflection of Point A across the x-axis. Note: x is the shared point and y differs only by sign.

~Point B is a reflection of Point A across the y-axis. Note: y is the shared point and x differs only by sign.

Reflections Across Both Axes

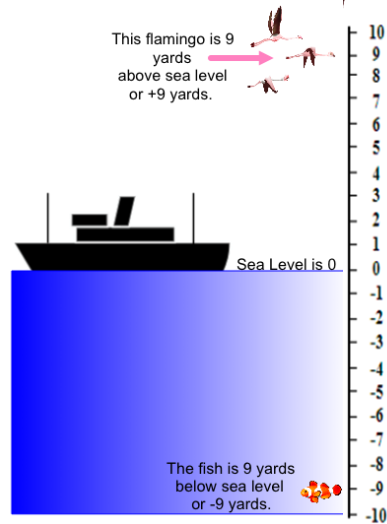
~Point C is a reflection of Point A across the x-axis, then the y-axis. Note: From Point C to Point B, x is the shared point and y differs only by sign. From Point B to Point A, y is the shared point and x differs only by sign.

~Point C is also a reflection of Point A across the y-axis, then the x-axis. Note: From Point C to Point D, y is the shared point and x differs only by sign. The next reflection occurs from Point D to Point A as x is the shared point and y differs only by sign.

Examples for Representing Zero in Real-World Contexts

Zero can represent various ideas contextually. For example, it can represent sea level when measuring elevation. It can also represent a balance between credits and debits. The balloon image demonstrates that zero represents an equal quantity or balance between positive and negative charges.

Thinking about what zero represents in a real-world situation allows students to identify quantitative relationships between numbers.



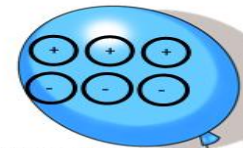
Put 5 dollars in savings



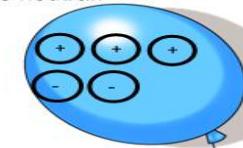
Spent 5 dollars for lunch =



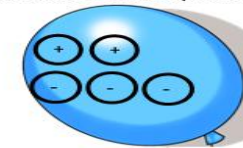
The net worth for the \$5.00 in savings and the \$5.00 spent is zero.



This balloon has no static charge. It is neutral.



This balloon has a positive static charge.



This balloon has a negative charge.

Potential Discussion Points for Helping Students Think About the Role of Zero in Real-World Contexts

SITUATION	NEGATIVE	ZERO	POSITIVE
Game/Sports: Golf/ Football	Below par / loss of yards	Par/line of scrimmage	Above par / gain of yards
Business	Loss (In the red)	Holding own	Profit (In the black)
Bank Accounts: Checkbooks	Charge- credit card Loans- interest paid / negative balance	Zero balance	Savings / Interest earned / Positive balance
Time and Time Zones	Past / Yesterday	Present / Midnight	Future / Tomorrow
Daylight Savings	Fall behind	Standard time	Spring ahead
Geologic or Historic Time	Before Common Era (B.C.)	Theoretical, but nonexistent year "0"	Common Era (C.E.)
Gauges/Dipsticks for Oil	Oil is low	Correct amount	Over filled
Tires	Flat	Correct pressure	Over inflated
Blood Pressure	Low blood pressure	Correct Pressure	High blood pressure
Eyes-Vision	-3.75	20/20	+3.75
Temperature-Vertical Time Line	Below Zero	Zero	Above zero
Elevation-Altitude	Below sea level	Sea level	Above sea level
Buildings	Basement / Lower levels	Ground floor	Attic / Upper floor

Apply and extend previous understandings of numbers to the system of rational numbers.

7. Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3^{\circ}\text{C} > -7^{\circ}\text{C}$ to express the fact that -3°C is warmer than -7°C .

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.

Number Line: Number lines provide a model for helping students interpret statements of equality. Numbers further right on the line are greater and numbers further left are smaller. For example $-3 > -7$ because -3 is further right than -7 on the number line. The greater than and less than arrows on each end are a vital component of the number line model and further identify the structure of the tool.



Interpreting and Writing Statements of Order: As students are able to make statements based on relative position on a number line as addressed above, they can then contextualize those numbers in a given situation. For example, a temperature of -7° will feel colder than -3° . Temperatures are growing greater or warmer the further right (or up) you travel on a number line. Temperatures are thus growing colder the further left (or down) you travel on the number line. Assigning a contextual “value” to either end of the number line such as *colder* or *warmer* will help students interpret statements of order.

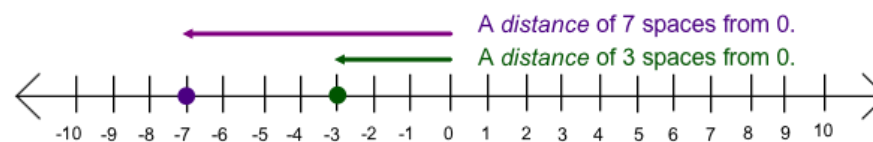
**Absolute Value:**

The confusing part of ordering and comparing integers for many students is the whole number association of quantity. -7 “feels” like more than -3 because 7 is greater than 3. Additionally, when comparing two debts, a debt of 7 dollars (-7) is more than a debt of 3 dollars. In that instance, the debt is more and yet -7 is still less than -3 . Therefore, it is helpful to be very clear about word choice when comparing integers. Greater and less than are used to compare the relative size of numbers. “More than” can be used to talk about the absolute value of a quantity, or its distance from zero. While it has many other functions, absolute value provides a way for discussing in what way an integer is more than another number.

Sione owes her mom \$7 and Jill owes her mom \$3.

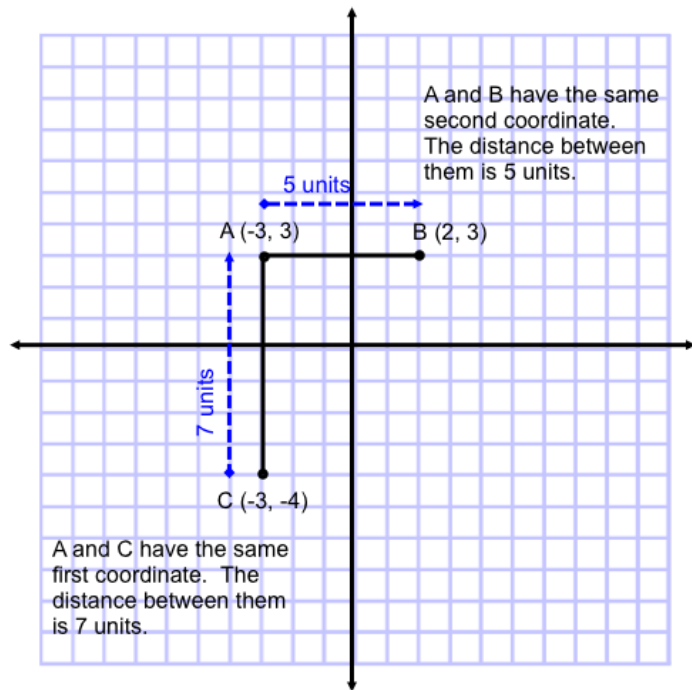
While -7 is a less than -3 , Sione’s debt is more than Jill’s. (Notice debt is often written using positive numbers, which makes Sione’s larger debt more evident.)

$$|-7| = 7 \text{ and } |-3| = 3$$



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8. Solve real world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.



Coordinate Pair: The first number in the pair provides the location on the x-axis. The second number in the pair provides the location on the y-axis. (Coordinate planes are two-dimensional and therefore need two coordinates to determine a position.)

Sharing the Same Coordinate: Whenever the first or second coordinates share the same number, they are on the same line. The coordinates can be thought of as street addresses. For example, *Billy lives on 17 West and 35 South (17, 35) and Ken lives on 17 West and 45 South (17, 45)*. Billy and Ken both live on the same street, or line: 17 West. This is evident in both pairs with 17 in the x-axis position.

Interpreting a Coordinate: Initially, students must plot points and draw line segments to develop an idea of when points are on the same line. However, as students become more familiar with the language of coordinate pairs, they should develop the ability to recognize when points are on the same line without having to plot the point and draw the line segment.

Using Absolute Value to Determine the Distance Between Two Points on a Line: The length of the line between two points can be determined by counting the spaces between the points. On the coordinate plane, Point A is 5 units away from Point B. Likewise, Point B is 5 units away from Point A. We wouldn't speak of them as being -5 units away, regardless of the direction traveled because distance is always positive. For example, if you traveled 2 miles to school and forgot your homework and had to go home to get it, you would have traveled another 2 miles for a total of 4 miles round trip. In essence, absolute value is the mathematical tool being used to compute distance between points.